B-math 2nd year Mid Semester Exam
Subject : Analysis III

Time : 3.00 hours Max.Marks 50.

1. a) Calculate the work done in moving a particle by a force $f(x,y) := (x^2 - y^2)\mathbf{i} + (2xy)\mathbf{j}$ once around the square bounded by the coordinate axes and the lines $x = a, y = a$ in the counter clockwise direction. (5)
b) Show that the vector field $f(x,y) := y\mathbf{i} - x\mathbf{j}$ is not a gradient. (10)

2. a) Let $(P(x,y), Q(x,y))$ be a given vector field on a domain $\Omega \subset \mathbb{R}^2$. Suppose there exists $f(.), g(.)$ such that

$$\frac{\partial P}{\partial y}(x,y) - \frac{\partial Q}{\partial x}(x,y) = f(x)Q(x,y) - g(y)P(x,y),$$

for $(x,y) \in \Omega$. Show that

$$u(x,y) := e^{\int_{x_0}^{x} f(s)\ ds + \int_{y_0}^{y} g(t)\ dt}$$

for $(x_0, y_0), (x,y) \in \Omega$ is an integrating factor of the the equation

$$P(x,y)\ dx + Q(x,y)\ dy = 0.$$

(10)

b) Solve the equation in a) with an integrating factor as above, when $P(x,y) := 2x^2 + y^2,\ Q(x,y) := 2x^3 - xy.$ (10)

3. a) For $Q := [0,1] \times [0,1]$. Define

$$f(x,y) = \left\{ \begin{array}{ccc} 3x + y & if & x = y \\ 0 & if & x \neq y \end{array} \right\}$$

Evaluate the integral $\int\int_Q f(x,y)\ dydx.$ (10)
b) Evaluate the integral $\int\int_Q f(x,y)\ dydx$ when $f(x,y) := sin(x+y)$ and $Q := [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}].$ (10)

4. Recall that the 'moment of inertia' of a region R in the plane about a line L is the (double)integral over R of $\delta^2(x, y)$ where

$$\delta(x, y) := \text{perpendicuar distance of (x, y)} \in \text{R from the line L.}$$

Let I_z denote the moment of inertia about the z-axis of a region R in the plane bounded by a simple closed curve ∂R. Show that there exists an integer n such that

$$nI_z = \int_{\partial R} x^3 dy - y^3 dx.$$

(10)